Estimations of total mass and energy of the universe

Dimitar Valev
Stara Zagora Department, Solar-Terrestrial Influences Laboratory, Bulgarian Academy of Sciences, 6000 Stara Zagora, Bulgaria

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Abstract

The recent astronomical observations indicate that the expanding universe is homogeneous, isotropic and asymptotically flat. The Euclidean geometry of the universe enables to determine the total gravitational and kinetic energy of the universe by Newtonian gravity in a flat space. By dimensional analysis, we have found the mass of the universe close to the Hoyle-Carvalho formula $M \sim c^3/(GH)$. This value is independent from the cosmological model and infers a size (radius) of the universe close to Hubble distance. It has been shown that almost the entire kinetic energy of the universe ensues from the cosmological expansion. Both, the total gravitational and kinetic energies of the universe have been determined in relation to an observer at an arbitrary location. The relativistic calculations for total kinetic energy have been made and the dark energy has been excluded from calculations. The total mechanical energy of the universe has been found close to zero, which is a remarkable result. This result supports the conjecture that the gravitational energy of the universe is approximately balanced with its kinetic energy of the expansion.

Key words: mass of the universe; energy of the universe; dimensional analysis; Newtonian gravity

1 Introduction

The problem for the average density of the universe $\overline{\rho}$ acquires significance when it has been shown that the General Relativity allows to reveal the geometry and evolution of the universe by simple cosmological models [1, 2, 3]. Crucial for the universe appears dimensionless total density $\Omega = \overline{\rho}/\rho_c$, where ρ_c is the critical density of the universe. In the case of $\Omega < 1$ (open universe) the global spatial curvature is negative and the geometry of the universe is hyperbolic and in the case of $\Omega > 1$ (closed universe) the curvature is positive and the geometry is spherical. In the special case of $\Omega = 1$ (flat universe) the curvature is zero and the geometry is Euclidean. Until recently scarce information has been available

about the density and geometry of the universe. The most trustworthy total matter density Ω has been determined by measurements of the dependence of the anisotropy of the Cosmic Microwave Background (CMB) upon the angular scale. The recent results show that $\Omega \approx 1 \pm \Delta\Omega$, where the error $\Delta\Omega$ decreases from 0.10 [4, 5] to 0.02 [6], i.e. the density of the universe is close to the critical one and the universe is asymptotically flat (Euclidean).

The fact that Ω is so close to a unit is not accidental since only at $\Omega = 1$ the geometry of the universe is flat and the flat universe was predicted from the inflationary theory [7]. The total density Ω includes matter density $\Omega_M = \Omega_b + \Omega_c$, where $\Omega_b \approx 0.05$ is density of baryon matter and $\Omega_c \approx 0.22$ is density of cold dark matter [8], and dark energy $\Omega_\Lambda \approx 0.73$ [9] producing an accelerating expansion of the universe [10, 11]. The found negligible CMB anisotropy $\delta T/T \sim 10^{-5}$ indicates that the early universe was very homogeneous and isotropic [12]. Three-dimensional maps of the distribution of galaxies corroborate homogeneous and isotropic universe on large scales greater than 100 Mps [13, 14].

Usually, Einstein pseudotensor is used for determination of the total energy of the universe [15, 16]. This approach is general for open, close and flat anisotropic models, but pseudotensorial calculations are dangerous as they are very coordinate dependent and thus, they may lead to ambiguous results [17]. Newtonian gravity still works reasonably in practically all gravitational problems, starting from Earth gravity, space flights and star systems and ending to birth of stars and star clusters, with exception of extremely compact objects like black holes and neutron stars possessing strong gravitational field causing non negligible curvature of the space. After recent CMB observations discovered that the global geometry of the universe is flat, some cosmological problems could be solved by Newtonian gravity in Euclidean space. This opportunity has been used in the paper to estimate total mechanical energy of the universe.

To determine gravitational and kinetic energy of the universe, information of the size and total mass of the universe are needed. There are different estimations of the mass of the universe covering very large interval from 3×10^{50} kg [18] to 1.6×10^{60} kg [19]. Also the estimations of the size (radius) of the universe are from 10 $Glyr$ [20] to more than of 78 $Glyr$ [21]. In the paper, we have found the mass of the observable universe by an original approach for cosmology, namely dimensional analysis.

2 Estimations of the total mass and size (radius) of the universe

Taking into account uncertainties of the estimations for the mass and size of the universe, an original approach for cosmology, namely dimensional analysis, has been applied below for estimation of the mass and size of the universe. The dimensional analysis is a conceptual tool often applied in physics to understand physical situations involving certain physical quantities. It is routinely used to check the plausibility of derived equations and computations. When the cer-

tain quantity, with which other determinative quantities would be connected, is known but the form of this connection is unknown, a dimensional equation was composed for its finding. Most often, the dimensional analysis is applied in mechanics where there are many problems having a few determinative quantities. The quantity of mass dimension in high energy physics is also obtained by means of the fundamental constants c, G and $\hbar$. This is the famous Planck mass $m_P \sim \sqrt{\hbar c/G} \approx 2.17 \times 10^{-8}\,\mathrm{kg}$, whose energy equivalent – the Planck energy $E_P = m_P c^2 \sim 10^{19}\,\mathrm{GeV}$ appears unification energy of the fundamental interactions.

The fundamental parameters as the gravitational constant G, speed of the light c and the Hubble constant $H \approx 70\ km\ s^{-1}\ Mps^{-1}$ [22] determine the global properties of the universe. Therefore, by means of these parameters, a mass dimension quantity m_x related to the universe could be constructed:

$$m_x = kc^{\alpha} G^{\beta} H^{\gamma} \tag{1}$$

where k is a dimensionless parameter of the order of magnitude of a unit and α, β and γ are unknown exponents which have been found by dimensional analysis.

Taking into account the dimensions of the quantities in formula (1) we obtain the system of linear equations for unknown exponents:

$$\begin{aligned} \alpha + 3\beta &= 0 \\ -\alpha - 2\beta - \gamma &= 0 \\ -\beta &= 1 \end{aligned} \tag{2}$$

The determinant Δ of the system is:

$$\Delta = \begin{vmatrix} 1 & 3 & 0 \\ -1 & -2 & -1 \\ 0 & -1 & 0 \end{vmatrix} = -1 \tag{3}$$

The determinant $\Delta \neq 0$, therefore the system has a unique solution. We find this solution by Kramer's formulae:

$$\begin{aligned} \alpha &= \frac{\Delta_1}{\Delta} = 3 \\ \beta &= \frac{\Delta_2}{\Delta} = -1 \\ \gamma &= \frac{\Delta_3}{\Delta} = -1 \end{aligned} \tag{4}$$

Thus, we find the mass m_x related to the universe:

$$m_x \sim \frac{c^3}{GH} \sim 10^{53}\,\mathrm{kg} \tag{5}$$

This value hits in the large interval for the mass of the universe mentioned at the end of Section 1 and coincides with Carvalho formula for the mass of the universe, deduced by totally different approach [23]. Thus, the quantity m_x, obtained by dimensional analysis by means of the fundamental parameters c, G and H, represents acceptable estimation of the mass of the universe. It is worthy to note that this value is independent from the cosmological model.

The universe is flat and the total density, including dark matter and dark energy, is $\overline{\rho} = \Omega\rho_c \approx \rho_c$, where the critical density of the universe ρ_c [24] determines from:

$$\rho_c = \frac{3H^2}{8\pi G} \approx 9.5 \times 10^{-27} kg\ m^{-3} \tag{6}$$

Since the universe is homogeneous and isotropic, it appears 3-dimensional homogeneous sphere for an observer at arbitrary location. From (5) and (6) we obtain:

$$\overline{\rho} = \Omega\rho_c = \frac{3\Omega H^2}{8\pi G} = \frac{m_x}{V} = \frac{3c^3}{4\pi R^3 GH} \tag{7}$$

From (7) we have estimated the size (radius) of the universe (R) close to the Hubble distance cH^{-1}:

$$R = (2/\Omega)^{1/3}\frac{c}{H} \sim cH^{-1} \tag{8}$$

The result, obtained from (8), shows our universe practically coincides with Hubble sphere.

The mass of the universe would be deduced more precisely by means of the recent density of the universe $\overline{\rho} = \Omega\rho_c = 3\Omega H^2/(8\pi G)$ and radius (size) of the universe $R \sim cH^{-1}$:

$$M = 4\pi R^3\overline{\rho}/3 \approx \frac{4\pi c^3\Omega\rho_c}{3H^3} = \frac{c^3\Omega}{2GH} \tag{9}$$

This expression is more accurate than (5), since the results of the dimensional analysis are correct with accuracy to a coefficient $k \sim 1$. This value practically coincides with the Fred Hoyle formula for the mass of the universe $M = c^3/(2GH)$ [25]. Any possible matter beyond the Hubble sphere does not affects the observer. Hence, it has no contribution in the mass and energy of the universe, calculated in relation to the observer.

Besides, we can estimate the total rest energy of the universe from (9) and Einstein equation:

$$E_0 = Mc^2 = \frac{c^5\Omega}{2GH} \tag{10}$$

3 Determination of the total mechanical energy of the universe

The results of dimensional analysis and CMB observations suggest that the universe appears homogeneous 3-dimensional sphere with radius R close to Hubble distance cH^{-1}. Hence, the gravitational potential energy U of the universe is:

$$U = -G\int_0^R \frac{M(r)dm}{r} = -\frac{16}{3}G\pi^2\overline{\rho}^2\int_0^R r^4 dr = -\frac{3GM^2}{5R} \tag{11}$$

where 0 is an arbitrary location of the observer, $R \sim cH^{-1}$ is the radius of the universe and $M(r) = 4\pi\overline{\rho}r^3/3$ is the mass of a sphere with radius r.

According to the equivalence of mass and energy, dark energy also possesses mass and gravitational energy. Replacing (8) and (9) in (11) we have found the total gravitational energy of the universe:

$$U = -\frac{3c^5\Omega^2}{20GH} \tag{12}$$

Similar approach has been used for calculation of the total gravitational energy of a body arising from gravitational interaction of the body with all masses of the universe [26, 27].

Taking into account formulae (10) and (12) we find $U = -\frac{3}{10}\Omega Mc^2$, i.e. the modulus of the total gravitational energy of the universe is close to 3/10 of its total rest energy.

The estimation of the total kinetic energy of the universe T is more complicated as a result of the diversity of movements of masses in the universe. We suggest that almost all kinetic energy of the universe is a result of the cosmological expansion since it includes movement of the enormous masses (galaxies and clusters of galaxies) with average speed of the order of magnitude of $c/2$. The rotation curves of galaxies show that the majority of stars move into the galaxies with speed less than $v_0 = 3 \times 10^5\ m\ s^{-1}$ [28]. Besides, on rare occasions, the peculiar (non-cosmological) velocities of galaxies exceed this value [29]. On the other hand, the speed of medium-distanced galaxies (and their stars), as a result of the cosmological expansion is of the order of magnitude of $c/2 = 1.5 \times 10^8\ m\ s^{-1}$. Obviously, the kinetic energy of an "average star" in the universe, ensuing from its peculiar movement, constitutes less than $(2v_0/c)^2 \sim 4 \times 10^{-6}$ part of its kinetic energy, ensuing from the cosmological expansion, therefore, former should be ignored.

Let us estimate the total kinetic energy of the universe in relation to an observer at arbitrary location. The total kinetic energy of the universe is the sum of the kinetic energy of all masses m_i moving in relation to the observer with speed v_i determined from Hubble law $v_i = Hr_i$, where $r_i \leq cH^{-1}$ is the distance between the observer and mass m_i placed within the Hubble sphere. Newtonian formula for kinetic energy $T = \frac{1}{2}\sum\limits_i m_i v_i^2$ was used in [30], but the distant masses recede from the observer with speeds comparable with the speed of the light c. Therefore, the relativistic formula for kinetic energy is used below:

$$T = c^2 \sum_i m_i [(1 - v_i^2/c^2)^{-1/2} - 1] \tag{13}$$

Since, for an arbitrary observer, the universe appears a 3-dimensional homogeneous sphere having radius $R \sim cH^{-1}$, the sum (13) can be replaced by the integral:

$$T = c^2 \int [(1 - v^2/c^2)^{-1/2} - 1] dm = 4\pi\overline{\rho}c^2 \int_0^R [(1 - v^2/c^2)^{-1/2} - 1] r^2 dr \tag{14}$$

Replacing $v \approx v_r$ with expression from Hubble law $v_r = Hr$, equation (14) transforms into:

$$T = 4\pi\overline{\rho}c^2 \int_0^R [(1 - H^2r^2/c^2)^{-1/2} - 1] r^2 dr = 4\pi\overline{\rho}c^2 I - \frac{4}{3}\pi\overline{\rho}c^2 R^3 \tag{15}$$

where $I = \int_0^R (1 - H^2r^2/c^2)^{-1/2} r^2 dr$.

The solution of the integral I is given from equation:

$$I = -\frac{cr}{2H}(c^2/H^2 - r^2)^{1/2} + \frac{c^3}{2H^3} \arcsin \frac{Hr}{c} \tag{16}$$

Considering (8) and replacing low and upper limits of integration we find:

$$I = \frac{\pi c^3}{4H^3} \tag{17}$$

Replacing (17) in (15) we obtain:

$$T = \frac{\pi^2\overline{\rho}c^5}{H^3} - \frac{4\pi\overline{\rho}c^5}{3H^3} \tag{18}$$

In consideration of $\overline{\rho} = \Omega\rho_c$ and (6) we find the total kinetic energy of the universe:

$$T = \frac{c^5\Omega}{2GH}(\frac{3\pi}{4} - 1) = Mc^2(\frac{3\pi}{4} - 1) \tag{19}$$

Taking into account (12) and (19), the total mechanical energy of the universe determines from:

$$E = T + U = \frac{c^5\Omega}{2GH}(\frac{3\pi}{4} - 1 - \frac{3\Omega}{10}) = Mc^2(\frac{3\pi}{4} - 1 - \frac{3\Omega}{10}) \approx 1.056Mc^2 \sim Mc^2 \tag{20}$$

Thus, the total mechanical energy of the universe is found close to its total rest energy. It is worthy to note that in the process of deducing the formula (19) the dark energy was accepted as involved in cosmological expansion ($\Omega = \Omega_M + \Omega_\Lambda$). But the dark energy has no kinetic energy, therefore it should be

excluded from calculations of the total kinetic energy of the universe. In result, the density $\overline{\rho} = \Omega\rho_c$ in formula (18) must be replaced with $\rho_M = \Omega_M\rho_c$ and taking into account formula (6) we find a relativistic formula for the total kinetic energy of the universe:

$$T = \frac{c^5\Omega_M}{2GH}(\frac{3\pi}{4} - 1) = Mc^2\Omega_M(\frac{3\pi}{4} - 1) \approx 1.356\Omega_M Mc^2 \tag{21}$$

The recent value of matter density is between $\Omega_M = 0.19$ [31] and $\Omega_M = 0.27$ [9]. As a result, the total kinetic energy of the universe $T \approx (0.26 \div 0.37)Mc^2$, i.e. close to $3/10$ of its total rest energy Mc^2.

Finally, from (12) and (21) we find the total mechanical energy of the universe:

$$E = T + U = \frac{c^5}{2GH}[\Omega_M(\frac{3\pi}{4} - 1) - \frac{3\Omega^2}{10}) = Mc^2[\frac{\Omega_M}{\Omega}(\frac{3\pi}{4} - 1) - \frac{3\Omega}{10}] \sim 0 \tag{22}$$

It is remarkably, that the total mechanical energy of the universe is close to zero. This result supports the conjecture that the gravitational energy of the universe is approximately balanced with its kinetic energy of the expansion [32]. According to formula (22), the total kinetic energy of the universe is strictly equal to zero in case of $\Omega_M = \frac{3}{10}\Omega^2/(3\pi/4 - 1) \approx 0.22$. This value should be discussed as a prediction of the suggested model, which the future more accurate observations are able to test.

4 Conclusions

The recent astronomical observations indicate that the expanding universe is homogeneous, isotropic and asymptotically flat. The Euclidean geometry of the universe enables to determine the total kinetic and gravitational energies of the universe by Newtonian gravity in a flat space.

By an original approach for cosmology, namely dimensional analysis, a mass dimension quantity of the order of 10^{53} kg, related to the universe, has been found close to Hoyle-Carvalho formula for the mass of the universe. This value is independent from the cosmological model and infers the size (radius) of the universe close to Hubble distance cH^{-1}.

Both, the total kinetic and gravitational energies of the universe have been determined in relation to an observer at arbitrary location. Based on the simple homogeneous and isotropic model of the flat universe which expands according to Hubble law, we have found equation for the total gravitational energy of the universe. The modulus of the total gravitational energy of the universe is estimated to $3/10$ of its total rest energy Mc^2.

The relativistic calculations for total kinetic energy have been made and the dark energy has been excluded from calculations. The total kinetic energy of the universe has been found close to the modulus of its total gravitational energy. Therefore, the total mechanical energy of the universe is close to zero, which is

a remarkable result. This result supports the conjecture that the gravitational energy of the universe is approximately balanced with its kinetic energy of the expansion.

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